1. **What is the value of** $\sqrt{25+10\sqrt{6}}+\sqrt{25-10\sqrt{6}}$?

A. $2\sqrt{5}$
B. $\sqrt{55}$
C. $2\sqrt{15}$
D. 50
E. 60

Square the given expression to get rid of the roots, though don't forget to un-square the value you get at the end to balance this operation and obtain the right answer:

Must know fro the GMAT: $(x+y)^2=x^2+2xy+y^2$ (while $(x-y)^2=x^2-2xy+y^2$).

So we get:
$=(25+10\sqrt{6})+2(\sqrt{25+10\sqrt{6}})(\sqrt{25-10\sqrt{6}})+(25-10\sqrt{6})$.

Note that sum of the first and the third terms simplifies to $(25+10\sqrt{6})+(25-10\sqrt{6})=50$, so we have $50+2(\sqrt{25+10\sqrt{6}})(\sqrt{25-10\sqrt{6}})$ -- > $50+2(\sqrt{25+10\sqrt{6}})(\sqrt{25-10\sqrt{6}})=50+2\sqrt{(25+10\sqrt{6})(25-10\sqrt{6})}$.

Also **must know for the GMAT:** $(x+y)(x-y)=x^2-y^2$, thus
$50+2\sqrt{(25+10\sqrt{6})(25-10\sqrt{6})}=50+2\sqrt{25^2-(10\sqrt{6})^2)}=50+2\sqrt{625-600}=50+2\sqrt{25}=60$.

Recall that we should un-square this value to get the right the answer: $\sqrt{60}=2\sqrt{15}$.

Answer: C.

2. **What is the units digit of** $(17^3)^4-1973^{3^2}$?

A. 0
B. 2
C. 4
D. 6
E. 8

Must know for the GMAT:

I. The units digit of $(abc)^n$ is the same as that of c^n, which means that the units digit of $(17^3)^4$ is that same as that of $(7^3)^4$ and the units digit of 1973^{3^2} is that same as that of 3^{3^2}.

II. If exponentiation is indicated by stacked symbols, the rule is to work from the top down, thus:
$a^{m^n}=a^{(m^n)}$ and not $(a^m)^n$, which on the other hand equals to a^{mn}.

So:
$(a^m)^n=a^{mn}$;

$a^{m^n}=a^{(m^n)}$.

Thus, $(7^3)^4=7^{(3*4)}=7^{12}$ and $3^{3^2}=3^{(3^2)}=3^9$.

III. The units digit of integers in positive integer power repeats in specific pattern (cyclicity): The units digit of 7 and 3 in positive integer power repeats in patterns of 4:

1. 7^1=7 (last digit is 7)
2. 7^2=9 (last digit is 9)

3. 7^3=3 (last digit is 3)
4. 7^4=1 (last digit is 1)
5. 7^5=7 (last digit is 7 again!)
...

1. 3^1=3 (last digit is 3)
2. 3^2=9 (last digit is 9)
3. 3^3=27 (last digit is 7)
4. 3^4=81 (last digit is 1)
5. 3^5=243 (last digit is 3 again!)
...

Thus th units digit of 7^{12} will be 1 (4th in pattern, as 12 is a multiple of cyclicty number 4) and the units digit of 3^9 will be 3 (first in pattern, as 9=4*2+**1**).

So, we have that the units digit of $(17^3)^4=17^{12}$ is 1 and the units digit of $1973^{32}=1973^9$ is 3. Also notice that **the second number is much larger then the first one**, thus **their difference will be negative**, something like 11-13=-2, which gives the final answer that the units digit of $(17^3)^4-1973^{3^2}$ is 2.

Answer B.

3. **If $5^{10x}=4{,}900$ and $2^{\sqrt{y}}=25$ what is the value of $\frac{(5^{(x-1)})^5}{4^{-\sqrt{y}}}$?**
A. 14/5
B. 5
C. 28/5
D. 13
E. 14

First thing one should notice here is that x and y must be some irrational numbers (4,900 has other primes then 5 in its prime factorization and 25 doesn't have 2 as a prime at all), so we should manipulate with given expressions rather than to solve for x and y.

$5^{10x}=4{,}900$ --> $(5^{5x})^2=70^2$ --> $5^{5x}=70$

Answer: E.

4. **What is the value of $5+4*5+4*5^2+4*5^3+4*5^4+4*5^5$?**
A. 5^6
B. 5^7
C. 5^8
D. 5^9
E. 5^10

This question can be solved in several ways:

Traditional approach: $5+4*5+4*5^2+4*5^3+4*5^4+4*5^5=5+4(5+5^2+5^3+5^4+5^5)$ Note that we have the sum of geometric progression in brackets with first term equal to 5 and common ratio also equal to 5. **The sum of the first n terms of geometric progression is given by: $sum=\frac{b*(r^n-1)}{r-1}$, where b is the first term, n # of terms and r is a common ratio $\neq 1$.**

So in our case: $5+4(5+5^2+5^3+5^4+5^5)=5+4(\frac{5(5^5-1)}{5-1})=5^6$.

30 sec approach based on answer choices:
We have the sum of 6 terms. Now, if all terms were equal to the largest term 4*5^5 we would

have: $sum=6*(4*5^5)=24*5^5\approx 5^2*5^5\approx 5^7$, so the actual sum must be less than 5^7, thus the answer must be A: 5^6.

Answer: A.

5. **If $x=23^2*25^4*27^6*29^8$ and is a multiple of 26^n, where n is a non-negative integer, then what is the value of $n^{26}-26^n$?**
A. -26
B. -25
C. -1
D. 0
E. 1

$23^2*25^4*27^6*29^8=odd*odd*odd*odd=odd$ so x is an odd number. The only way it to be a multiple of 26^n (even number in integer power) is when $n=0$, in this case $26^n=26^0=1$ and 1 is a factor of every integer.
Thus $n=0$ --> $n^{26}-26^n=0^{26}-26^0=0-1=-1$. **Must know for the GMAT**: $a^0=1$, for $a\neq 0$ - any nonzero number to the power of 0 is 1. **Important note: the case of 0^0 is not tested on the GMAT.**

Answer: C.

6. **If $x=\sqrt[5]{-37}$ then which of the following must be true?**
A. $\sqrt{-x}>2$
B. x>-2
C. x^2<4
D. x^3<-8
E. x^4>32

Must know for the GMAT: Even roots from negative number is undefined on the GMAT (as GMAT is dealing only with Real Numbers): $\sqrt[even]{negative}=undefined$, for example $\sqrt{-25}=undefined$.

Odd roots have the same sign as the base of the root. For example, $\sqrt[3]{125}=5$ and $\sqrt[3]{-64}=-4$.

Back to the original question:

As $-2^5=-32$ then x must be a little bit **less** than -2 --> $x=\sqrt[5]{-37}\approx -2.1<-2$.
Thus $x^3\approx(-2.1)^3\approx -8.something<-8$, so option D must be true.

As for the other options:
A. $\sqrt{-x}=\sqrt{-(-2.1)}=\sqrt{2.1}<2$, $\sqrt{-x}>2$ is not true.
B. $x\approx -2.1<-2$, thus x>-2 is also not true.
C. $x^2\approx(-2.1)^2=4.something>4$, thus x^2<4 is also not true.
E. $x^4\approx(-2.1)^4\approx 17$, (2^4=16, so anyway -2.1^4 can not be more than 32) thus x^4>32 is also not true.

Answer: D.

7. **If** $x=\sqrt{10}+\sqrt[3]{9}+\sqrt[4]{8}+\sqrt[5]{7}+\sqrt[6]{6}+\sqrt[7]{5}+\sqrt[8]{4}+\sqrt[9]{3}+\sqrt[10]{2}$**, then which of the following must be true:**

A. x<6
B. 6<x<8
C. 8<x<10
D. 10<x<12
E. x>12

Here is a little trick: any positive integer root from a number more than 1 will be more than 1. For example: $\sqrt[1000]{2}>1$.

Now, $\sqrt{10}>3$ (as 3^2=9) and $\sqrt[3]{9}>2$ (2^3=8).
Thus $x=(\#\ more\ then\ 3)+(\#\ more\ then\ 2)+(7\ numbers\ more\ then\ 1)=$
$=(\#\ more\ then\ 5)+(\#\ more\ then\ 7)=$
$=(\#\ more\ then\ 12)$

Answer: E.

8. **If** x **is a positive number and equals to** $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+...}}}}$**, where the given expression extends to an infinite number of roots, then what is the value of x?**

A. $\sqrt{6}$
B. 3
C. $1+\sqrt{6}$
D. $2\sqrt{3}$
E. 6

Given: $x>0$ and $x=\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+...}}}}$ --> $x=\sqrt{6+(\sqrt{6+\sqrt{6+\sqrt{6+...}}})}$, as the expression under the square root extends infinitely then expression in brackets would equal to x itself and we can safely replace it with x and rewrite the given expression as $x=\sqrt{6+x}$. Square both sides: $x^2=6+x$ --> $(x+2)(x-3)=0$ --> $x=-2$ or $x=3$, but since $x>0$ then: $x=3$.

Answer: B.

9. **If** x **is a positive integer then the value of** $\frac{22^{22x}-22^{2x}}{11^{11x}-11^{x}}$ **is closest to which of the following?**

A. 2^{11x}
B. 11^{11x}
C. 22^{11x}
D. $2^{22x}*11^{11x}$
E. $2^{22x}*11^{22x}$

Note that we need *approximate* value of the given expression. Now, 22^{22x} is much larger number than 22^{2x}. Hence $22^{22x}-22^{2x}$ will be very close to 22^{22x} itself, basically 22^{2x} is negligible in this case. The same way $11^{11x}-11^{x}$ will be very close to 11^{11x} itself.

Thus .

You can check this algebraically as well: $\frac{22^{22x}-22^{2x}}{11^{11x}-11^{x}}=\frac{22^{2x}(22^{20x}-1)}{11^{x}(11^{10x}-1)}$. Again, -1, both in denominator and nominator is negligible value and we'll get the same expression as above:

Answer: D.

10. **Given that** $5x=125-3y+z$ **and** $\sqrt{5x}-5-\sqrt{z-3y}=0$**, then what is the value of** $\sqrt{\frac{45(z-3y)}{x}}$**?**
A. 5
B. 10
C. 15
D. 20
E. Can not be determined

Rearranging both expressions we'll get: $5x-(z-3y)=125$ and $\sqrt{5x}-\sqrt{z-3y}=5$. Denote $\sqrt{5x}$ as a and $\sqrt{z-3y}$ as b.

So we have that $a^2-b^2=125$ and $a-b=5$. Now, $a^2-b^2=(a-b)(a+b)=125$ and as $a-b=5$ then $(a-b)(a+b)=5*(a+b)=125$ --> $a+b=25$. Thus we get two equations with two unknowns: $a+b=25$ and $a-b=5$ --> solving for a --> $a=15=\sqrt{5x}$ --> $x=45$. Solving for b --> $b=10=\sqrt{z-3y}$

Finally, $\sqrt{\frac{45(z-3y)}{x}}=\sqrt{\frac{45*10^2}{45}}=10$.

Answer: B.

11. If $x>0$, $x^2=2^{64}$ **and** $x^x=2^y$ **then what is the value of** y**?**
A. 2
B. 2^(11)
C. 2^(32)
D. 2^(37)
E. 2^(64)

$x^2=2^{64}$ --> $x=\sqrt{2^{64}}=2^{\frac{64}{2}}=2^{32}$ (note that $x=-\sqrt{2^{64}}$ is not a valid solution as given that $x>0$).

Second step: $x^x=(2^{32})^{(2^{32})}=2^{32*2^{32}}=2^{2^5*2^{32}}=2^{2^{37}}=2^y$ --> $y=2^{37}$.

OR second step: $x^x=(2^{32})^x=2^{32x}=2^y$ --> $y=32x$ --> since $x=2^{32}$ then $y=32x=32*2^{32}=2^5*2^{32}=2^{37}$.

Answer: D.